

An aerial photograph of a forest landscape. A dirt road winds through the trees, and a stream flows through the lower right portion of the image. The forest is dense and green, with some areas appearing more open or cleared.

What is 3PG?

Adaptation of a presentation from Peter Sands

An overview of Landsberg and Waring's
model of forest productivity

3PG is:

- ✓ A tree growth model based on **P**hysiological **P**inciples that **P**redict **G**rowth
- ✓ Bridges the gap between mensuration-based growth and yield models and process-based, C-balance models
- ✓ Provides fully dynamic predictions of biomass pools, stand attributes, stocking and soil water usage
- ✓ Maintains an admirable level of simplicity

Comparison with empirical models

✓ Advantages

- based on a wide range of conditions
- applicable under changing conditions, “at the edges”, to novel situations
- provides explanation, aids understanding

✓ Disadvantages

- not as widely understood as empirical growth models
- not necessarily as accurate, either
- can require data not readily available

A quick summary of 3-PG

Attribute	Comments
Model type	Dynamic, includes process-based and empirical relationships
Time step	Month
Processes	NPP, biomass allocation, water usage and soil water balance, stem mortality, litterfall and root turnover
Inputs	Monthly climate data, soil texture and water capacity, fertility
Outputs	Biomass pools, stocking, available soil water, NPP and ET, average DBH and standard stand attributes, and others
Strengths	Fully dynamic, can be adapted for a range of species, provides management-related outputs
Weaknesses	Naïve treatment of soil nutrition, allocation based largely on size, poor predictor of canopy development and of mortality

Input data for 3-PG is ...

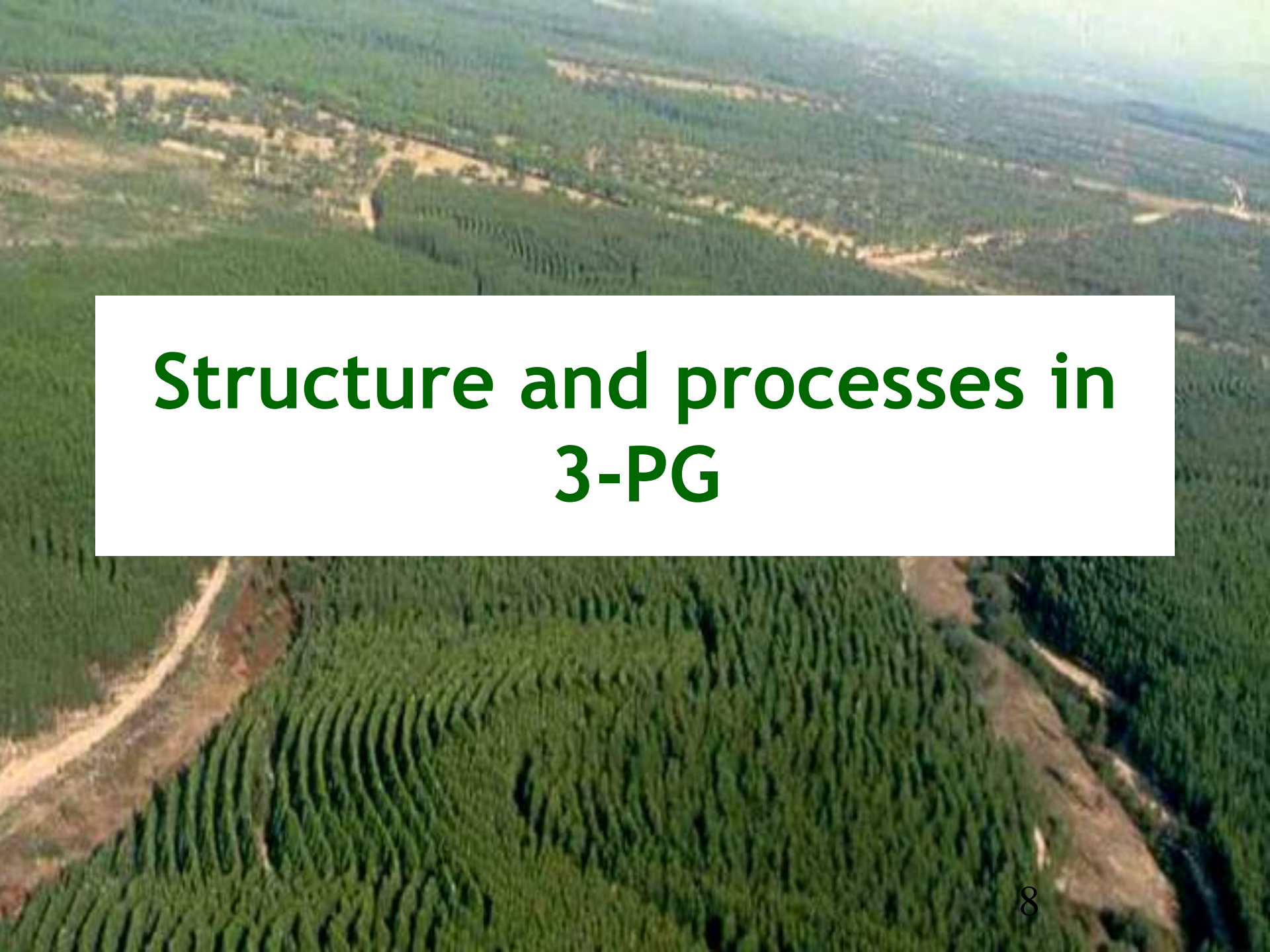
- ✓ ...of a quality and quantity that is readily obtained by the forest manager
 - mean monthly weather data
 - very basic physical site and soil factors
 - simple (naïve?) ranking of site fertility

Input data (continued)

- ✓ Climate data
 - monthly mean temperature, radiation, rainfall, VPD
 - observed or long-term average data
- ✓ Site and soil descriptors
 - latitude
 - soil texture and water capacity
 - fertility rating
- ✓ Also need stand initialisation data
 - foliage, stem and root biomass
 - stocking

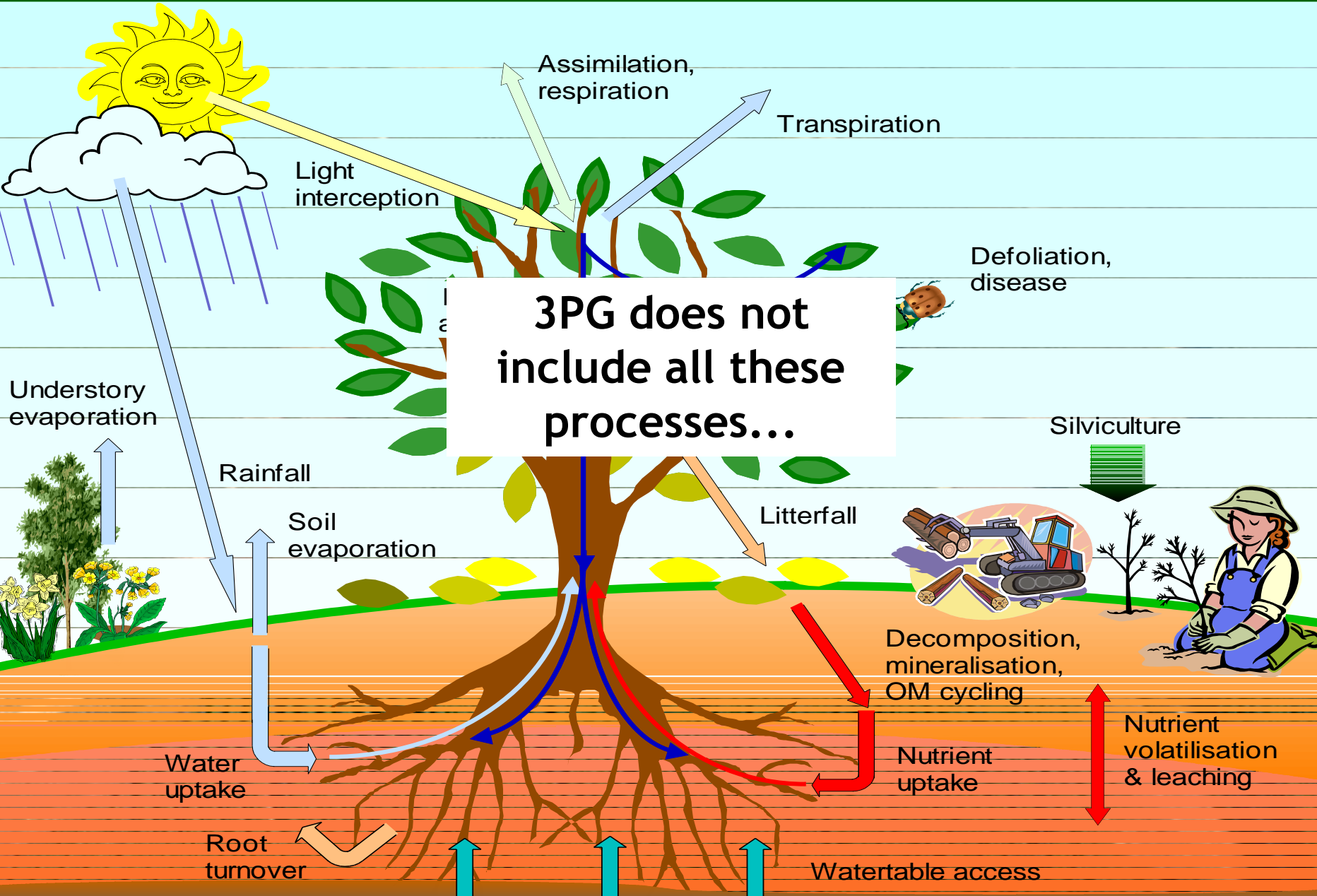
Main Components of 3-PG

- ✓ **Production of biomass** - environmental modification of light use efficiency; constant ratio of NPP to GPP
- ✓ **Biomass allocation** - affected by growing conditions and tree size
- ✓ **Stem mortality** - probability of death; self-thinning
- ✓ **Soil water balance** - single soil layer model; evapotranspiration determined from Penman-Monteith equation
- ✓ **Stand properties** - from biomass pools and assumptions about specific leaf area, branch+bark fraction, and wood density

An aerial photograph of a vast agricultural plantation, likely a rubber or palm oil estate. The image shows a dense grid of young trees planted in neat, parallel rows. A dirt road or path winds through the plantation, and the overall landscape is a mix of green and brownish-yellow, suggesting a semi-arid or tropical environment. The text "Structure and processes in 3-PG" is overlaid in a white box with green text.

Structure and processes in 3-PG

Conceptual PBM of forest growth



Conceptual PBM... (continued)

- ✓ McMurtrie and Wolf (1983) model is a common basis for many implementations of the conceptual model
- ✓ 3-PG follows this model
- ✓ W_i - biomass, η_i - proportion allocated
 γ_i - turnovers (litterfall, death, root turn.)
 l - leaves, s - stem, r - roots

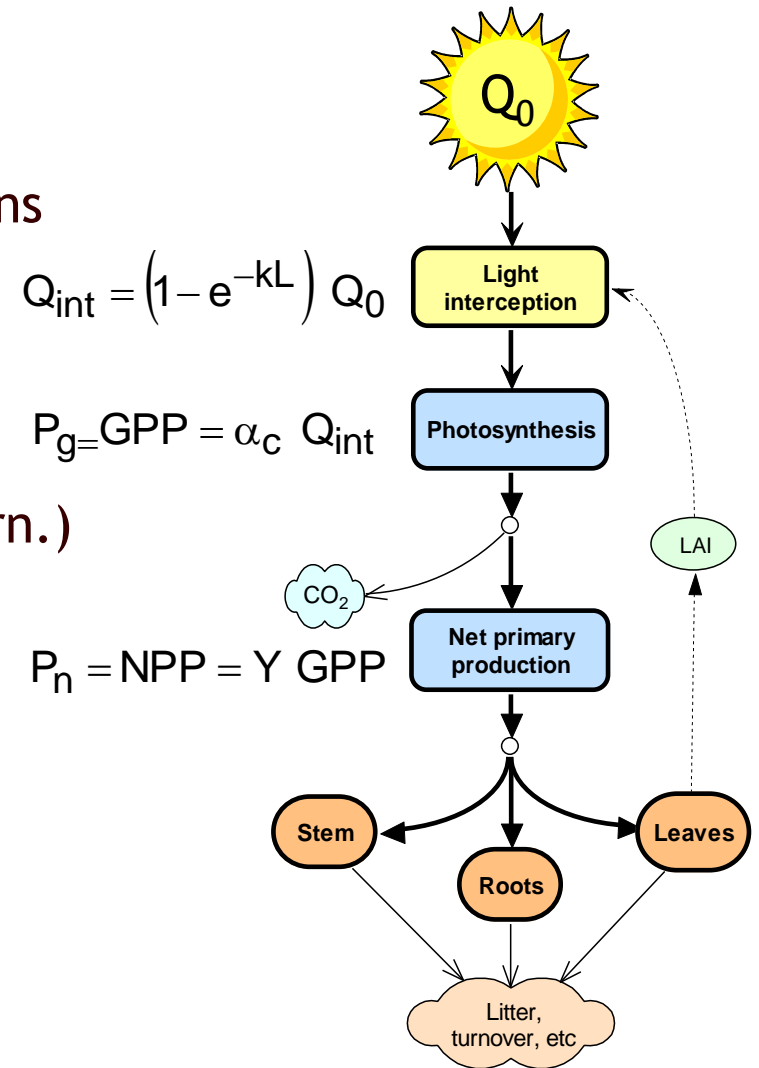
$$\Delta W_F = \eta_F P_n - \gamma_f W_F$$

$$\Delta W_l = \eta_l P_n - \gamma_l W_l$$

$$\Delta W_s = \eta_s P_n - \gamma_s W_s$$

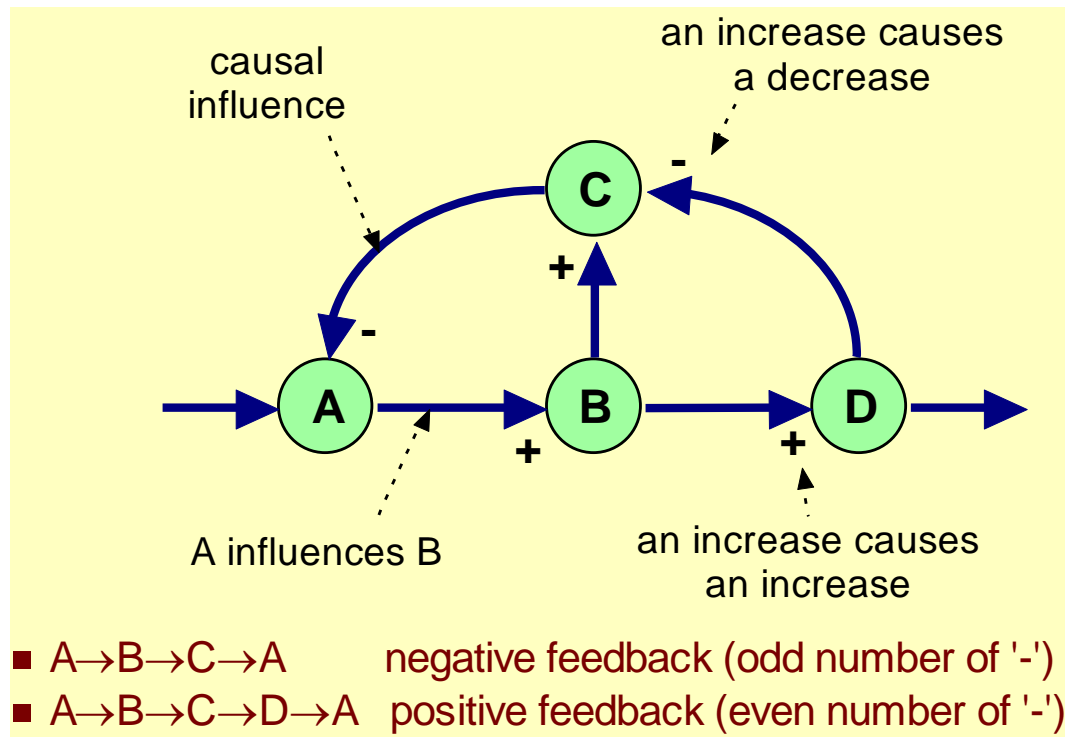
$$\Delta W_r = \eta_r P_n - \gamma_r W_r$$

$$\eta_l + \eta_s + \eta_r = 1$$



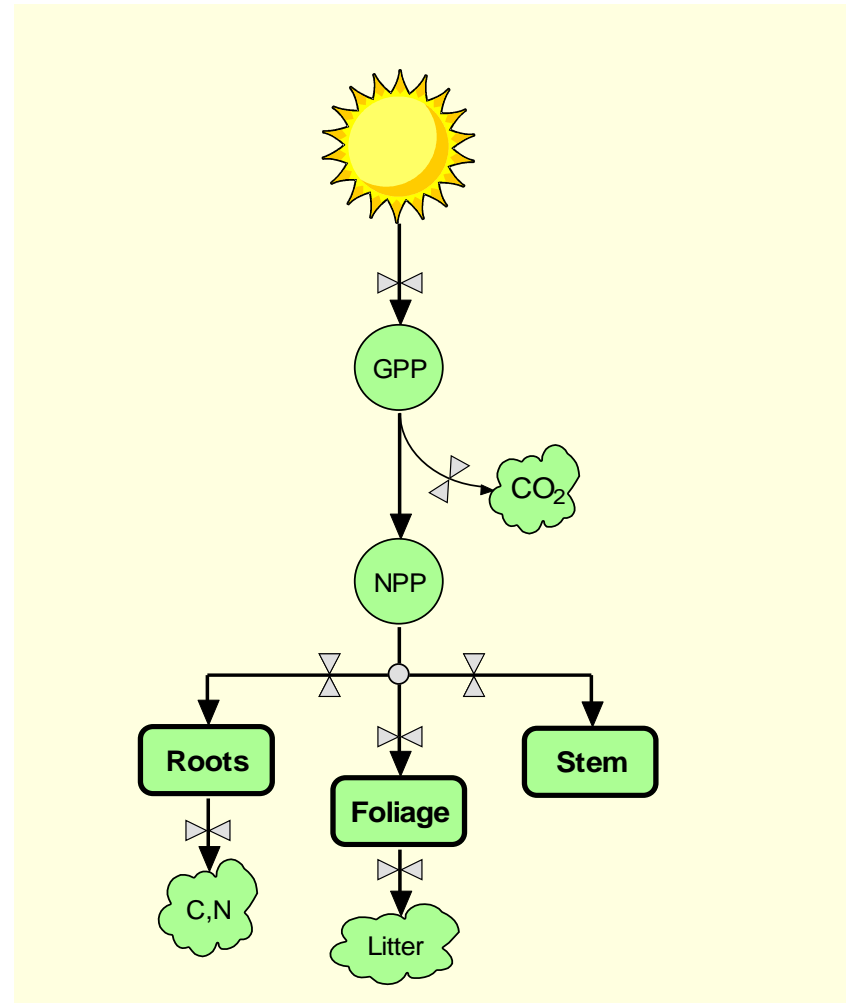
Causal loop diagrams

- ✓ Powerful tools to communicate and explore system behaviour
- ✓ They summarize structure, causal influences and feedback loops



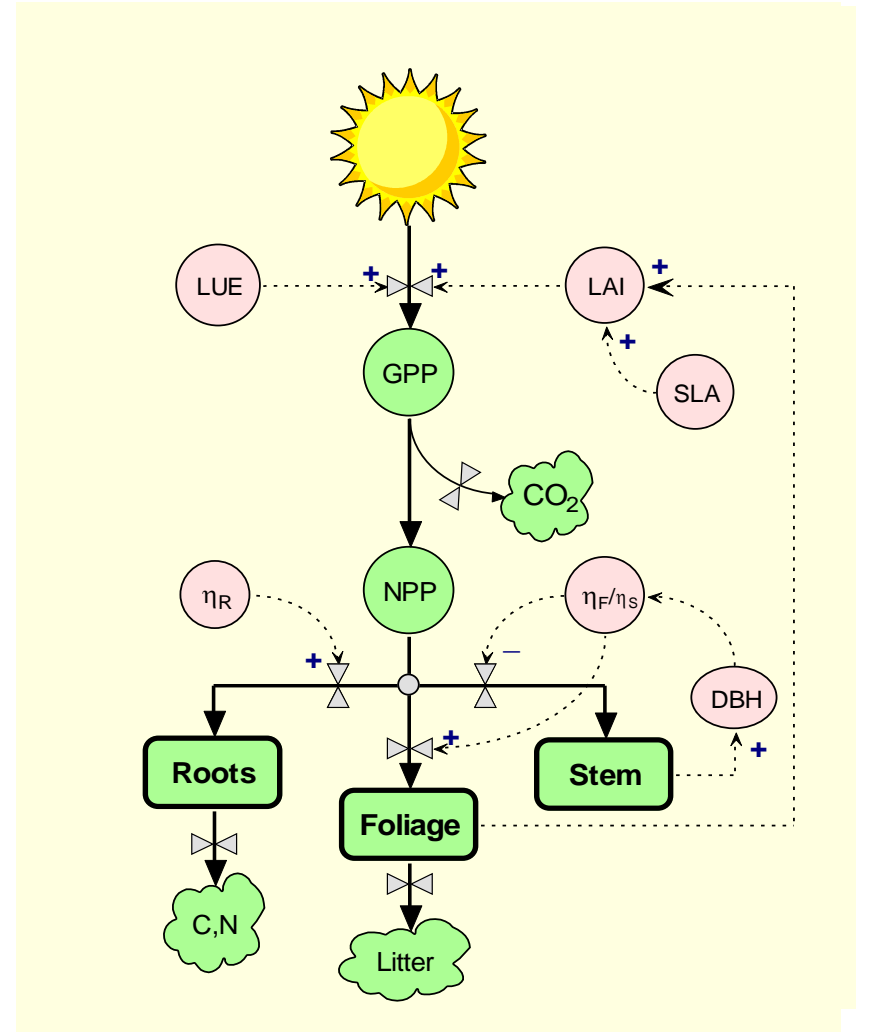
3-PG as a carbon flow model

- ✓ 3-PG is essentially a McMurtrie and Wolf (1983) carbon balance model
 - radiation is intercepted by the canopy
 - converted to assimilates
 - allocated to foliage, stem and roots
 - lost to respiration, litterfall and root turnover



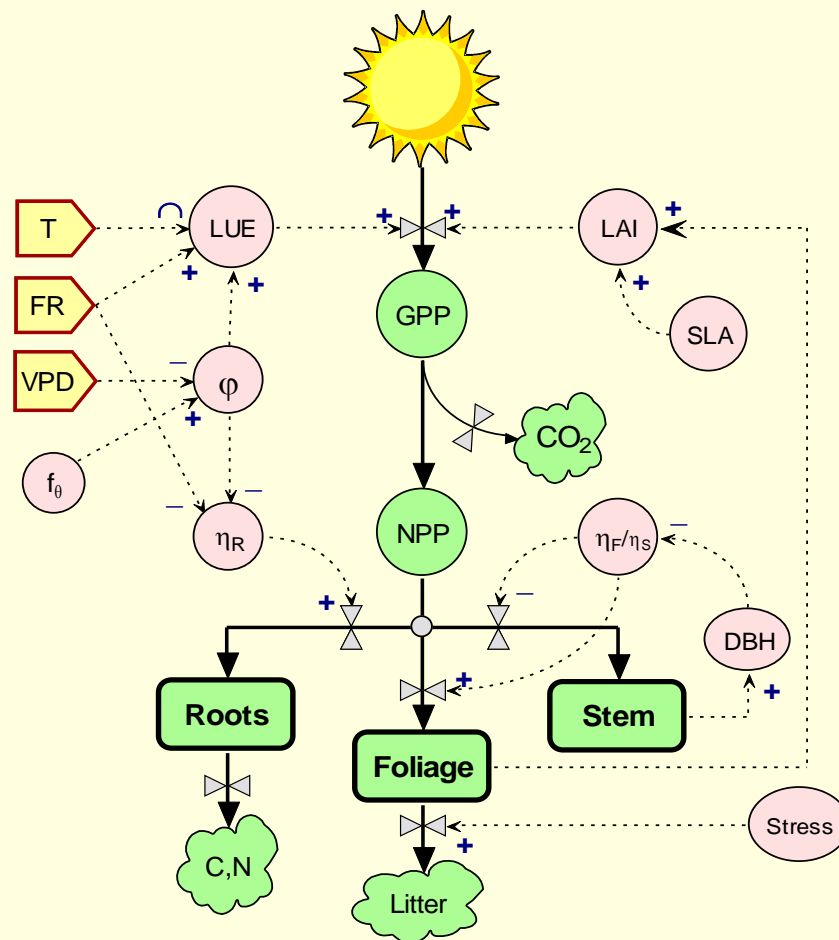
Assimilation and allocation

- ✓ Assimilation and allocation are based on simple, well established principles and sound observations
 - radiation interception via Beers law
 - assimilation via light use efficiency
 - simple foliage, stem and root allocation ratios
 - foliage:stem allocation depends on tree size



Site and environmental effects

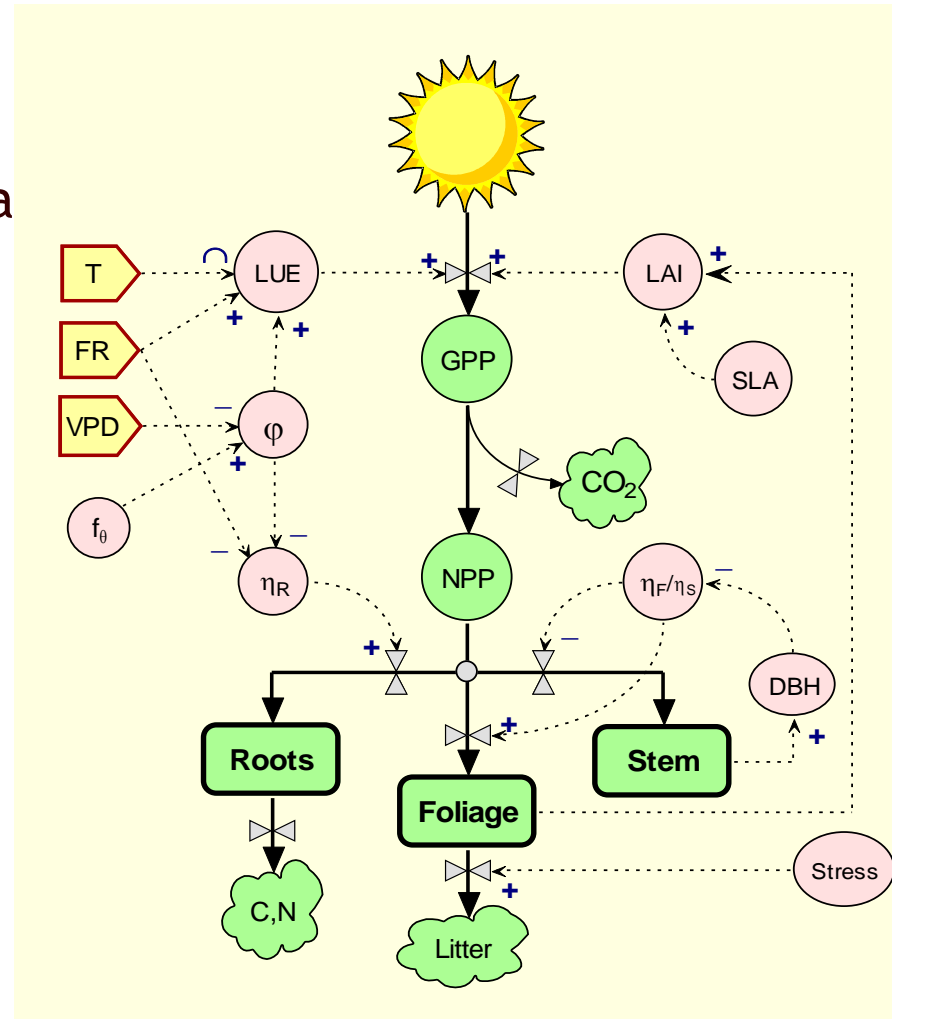
- ✓ Site and environmental factors affect growth (and water use) via simple empirical modifiers
 - temperature affects only LUE
 - VPD and soil water affect LUE and root allocation
 - site fertility affects root allocation and maybe LUE



Soil water balance

✓ Soil water balance via simple single layer model with transpiration determined using a Penman-Monteith equation

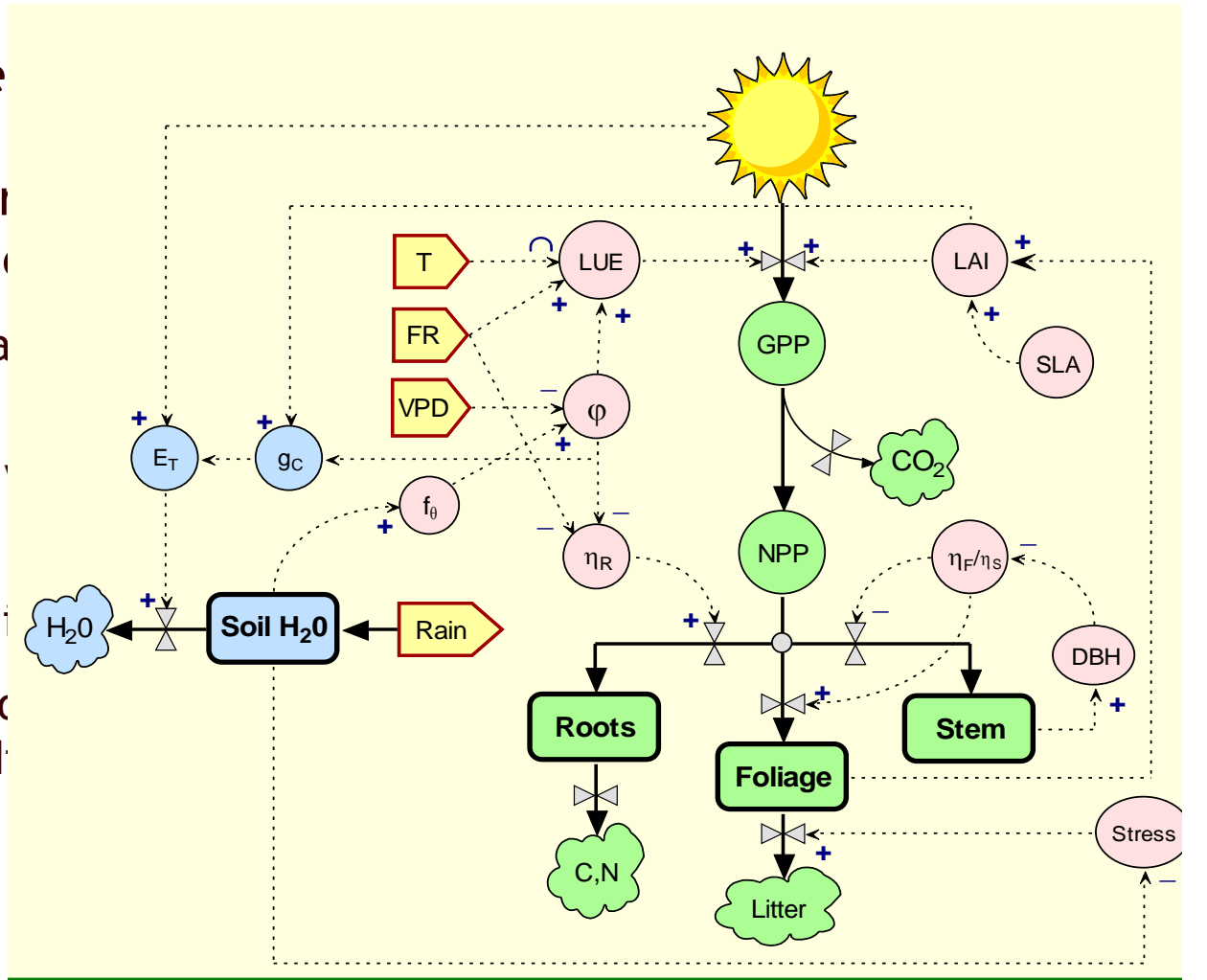
- canopy conductance scaled for canopy LAI
- and affected by VPD and soil water
- ET driven by radiation
- feedback from soil water status into growth modifiers



Soil water balance

✓ Soil water balance
single layer model
transpiration deter
Penman-Monteith

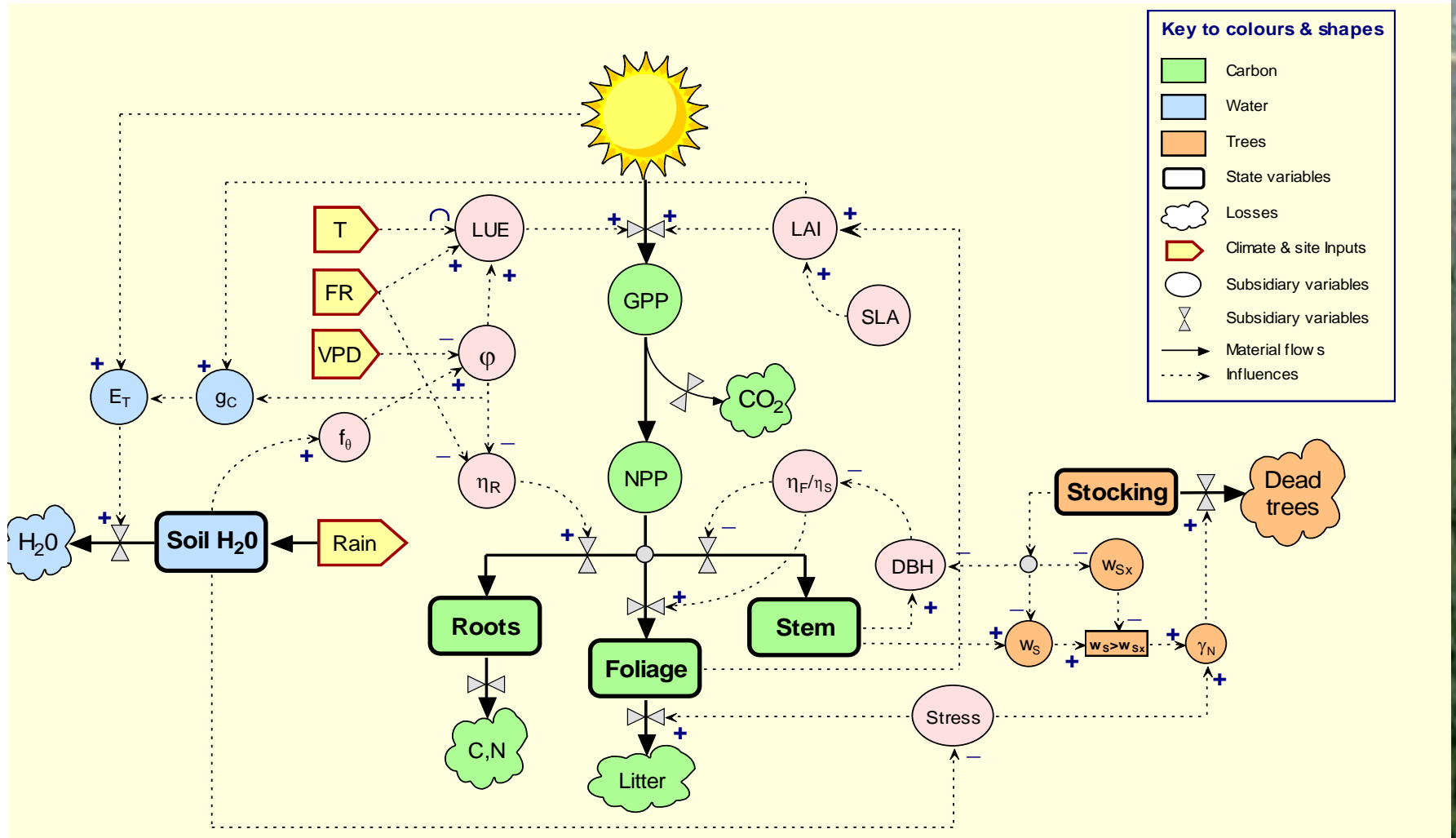
- canopy conductance
canopy LAI
- and affected by
water
- ET driven by radi
- feedback from so
into growth mod



Stocking and mortality

- ✓ Stocking is an essential component of 3-PG as it affects allocation through stand-mean DBH
 - mortality model very simple-minded
 - probability of death age and (potentially) stress related
 - density dependent mortality implemented via self-thinning law

This is the final “picture”





3-PG in more detail

Light interception

- ✓ Light is absorbed as it passes through canopy
- ✓ Intercepted radiation varies with LAI via Beer's law:

$$Q_{\text{int}} = (1 - e^{-kL}) Q_0$$

- ✓ LAI determined by specific leaf area (sla) and foliage biomass

$$\text{LAI} = W_F \text{ sla}$$

Light interception

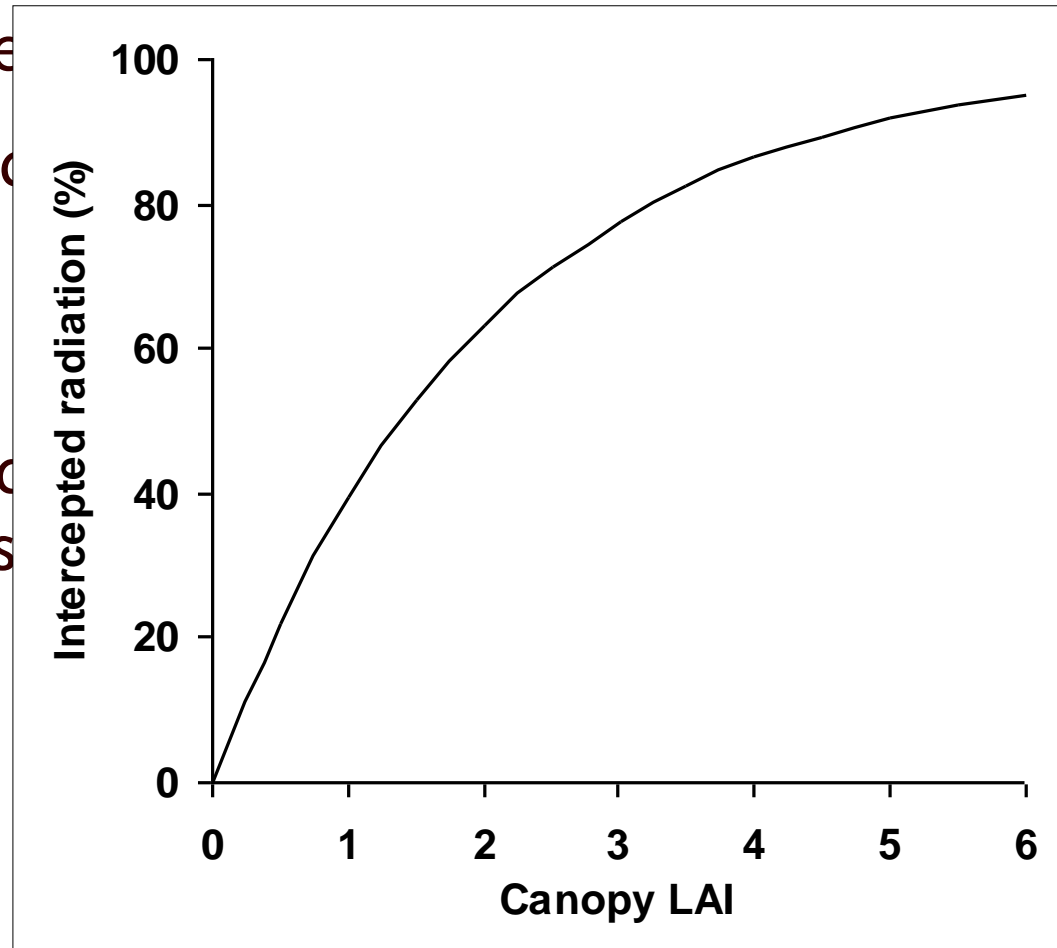
✓ Light is absorbed

✓ Intercepted radiation follows Beer-Lambert law:

$$Q_{\text{int}} = (1 - e^{-kL}) Q_0$$

✓ LAI determined from foliage biomass

$$\text{LAI} = W_F \text{ sla}$$



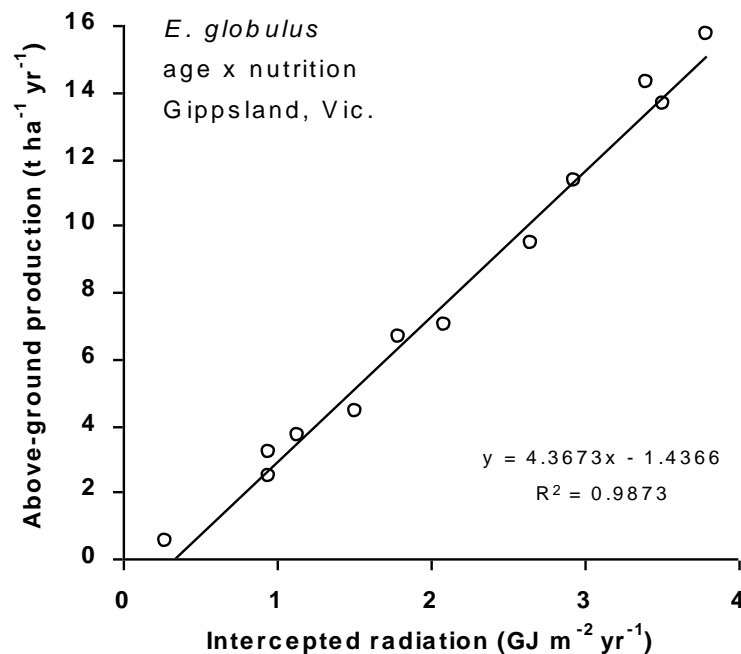
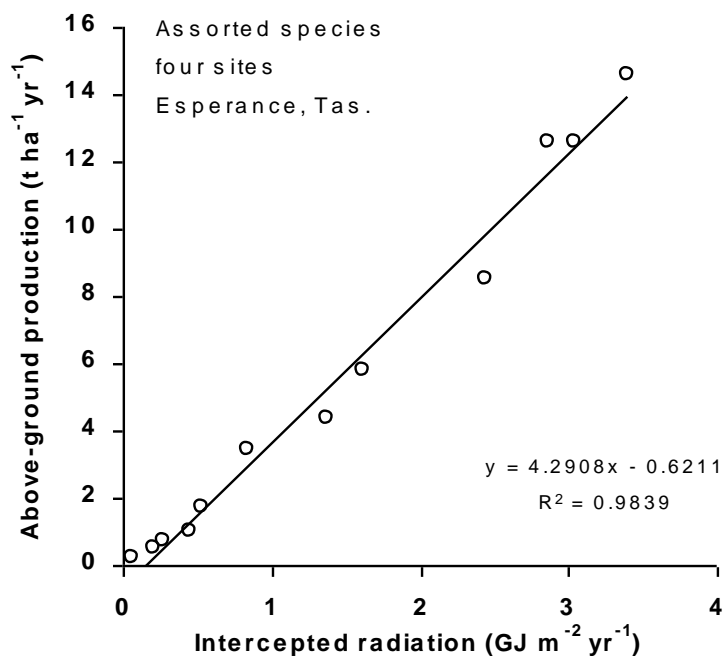
Production and solar radiation

- ✓ Observation shows
 - above-ground and gross production are linearly related to intercepted radiation
- ✓ Slope of these relationships is a measure of light use efficiency (LUE)
 - daily canopy-level LUE varies seasonally
 - annual stand-level LUE stable

Production and solar radiation

✓ Observation shows

- above-ground and gross production are linearly related



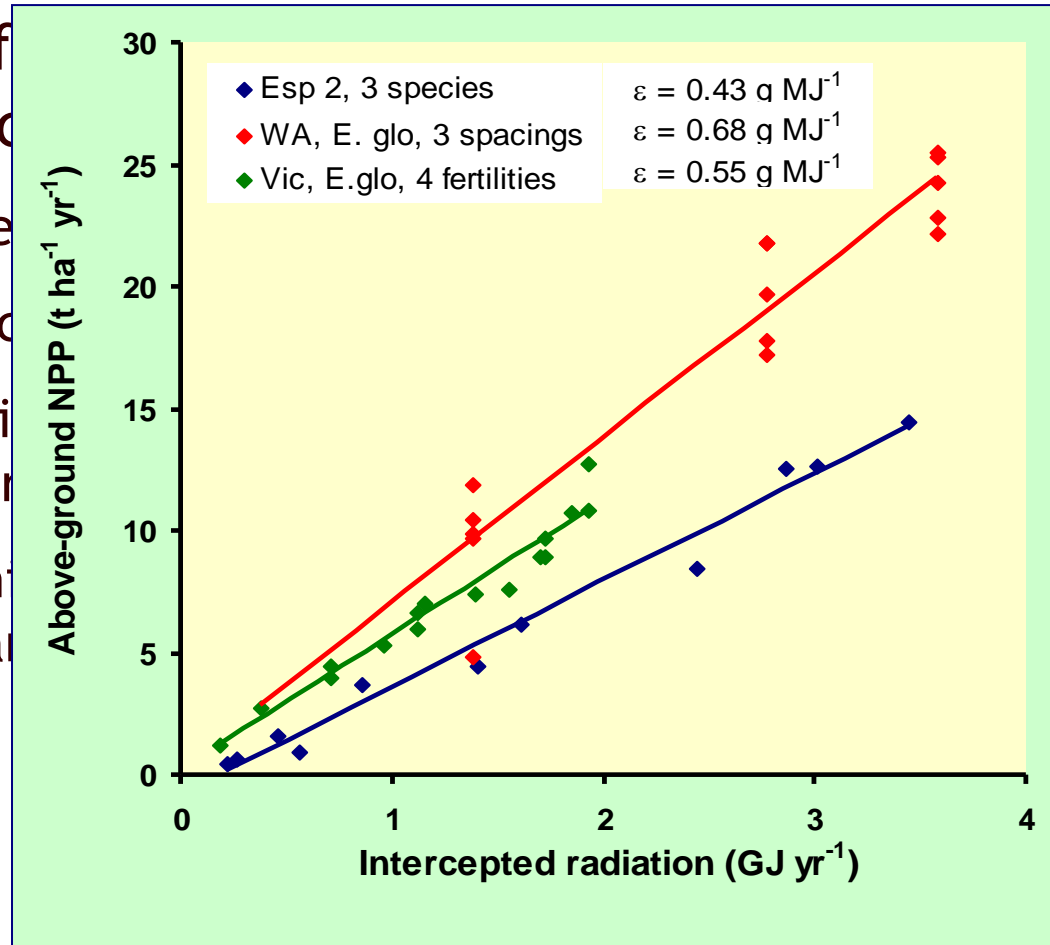
Light use efficiency

- ✓ Light use efficiency (LUE) is a powerful, simplifying concept
 - annual stand-level LUE quite stable
 - species-specific
 - varies with climatic and site factors through use of simple modifiers
 - early use of this concept by Fitzpatrick and Nix (1970) in GROWEST, and by Monteith (1972)

Light use efficiency

✓ Light use efficiency
simplifying conditions

- annual stand-level
- species-specific
- varies with climate
simple modifiers
- early use of the term
in GROWEST, and



Gross primary production

- ✓ Use of LUE is a key simplification in 3-PG
 - also known as “canopy quantum efficiency” denoted by α_c
 - GPP proportional to intercepted radiation:

$$P_g = \alpha_c (1 - e^{-kL}) Q_0$$

- α_c depends on site and climatic conditions

$$\alpha_c = f_T f_F f_N \min\{f_D, f_\theta\} f_{age} \alpha_{Cx}$$

Gross primary production

✓ Use of LUE is a key simplification

- also known as “canopy quantum yield”

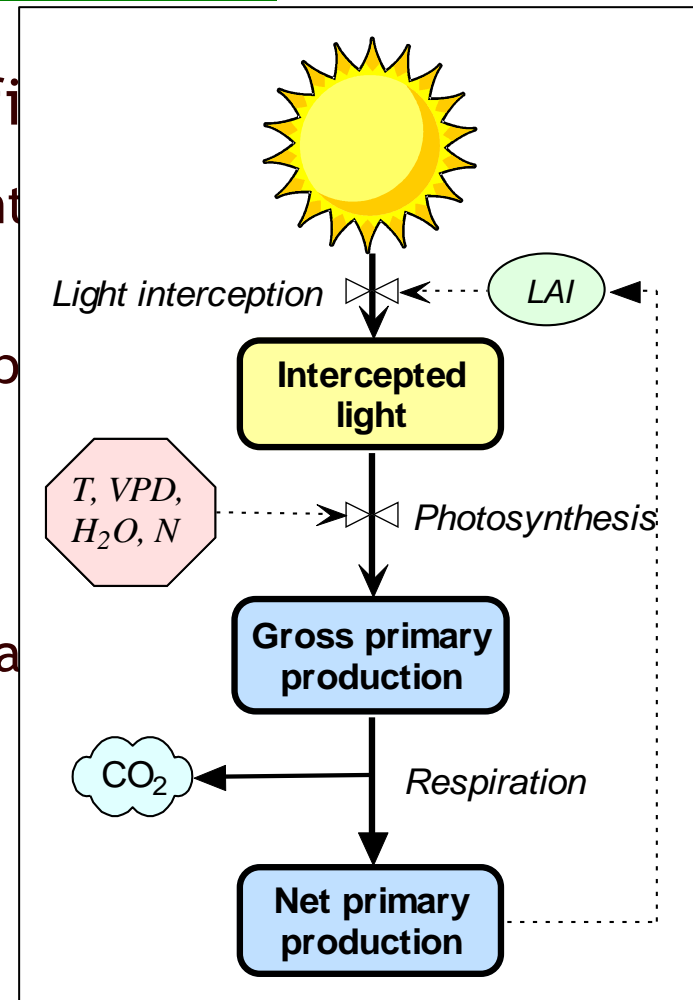
α_c

- GPP proportional to intercepted light

$$P_g = \alpha_C (1 - e^{-kL}) Q_0$$

- α_c depends on site and climate

$$\alpha_C = f_T f_F f_N \min\{f_D, f_\theta\} f_{age} \alpha_{Cx}$$



Net primary production

✓ 3-PG assumes constant fraction Y ($=0.47$) of GPP is lost as construction and maintenance respiration

✓ Net primary production is then

$$P_n = Y P_g = \alpha_c Y (1 - e^{-kL}) Q_0$$

✓ Y probably varies seasonally with temperature

- this would be an issue for a daily version of 3-PG

3-PG growth modifiers

Each environmental factor is represented by a growth modifier, i.e. a function of the factor which varies between 0 (total limitation) and 1 (no limitation)

Factor	Modifier	Parameters
Vapor pressure deficit	$f_D(D)$	k_D
Soil water	$f_\theta(\theta)$	$\theta_{max}, c_\theta, n_\theta$
Temperature	$f_T(T_{av})$	$T_{min}, T_{opt}, T_{max}$
Frost	$f_F(d_f)$	k_F
Site nutrition	$f_N(FR)$	f_{N0}
Stand age	$f_{age}(t)$	n_{age}, r_{age}

Effects on production

- ✓ All modifiers affect canopy production:

$$\alpha_C = f_T f_F f_N \min\{f_D, f_\theta\} f_{age} \alpha_{Cx}$$

Where α_C is maximum canopy quantum efficiency

- ✓ In 3-PG the combination of modifiers called “PhysMod “

$$\varphi = \min\{f_D, f_\theta\} f_{age}$$

also affects canopy conductance

Temperature growth modifier

$$f_T(T_a) = \left(\frac{T_a - T_{min}}{T_{opt} - T_{min}} \right) \left(\frac{T_{max} - T_a}{T_{max} - T_{opt}} \right)^{(T_{max} - T_{opt}) / (T_{opt} - T_{min})}$$

where

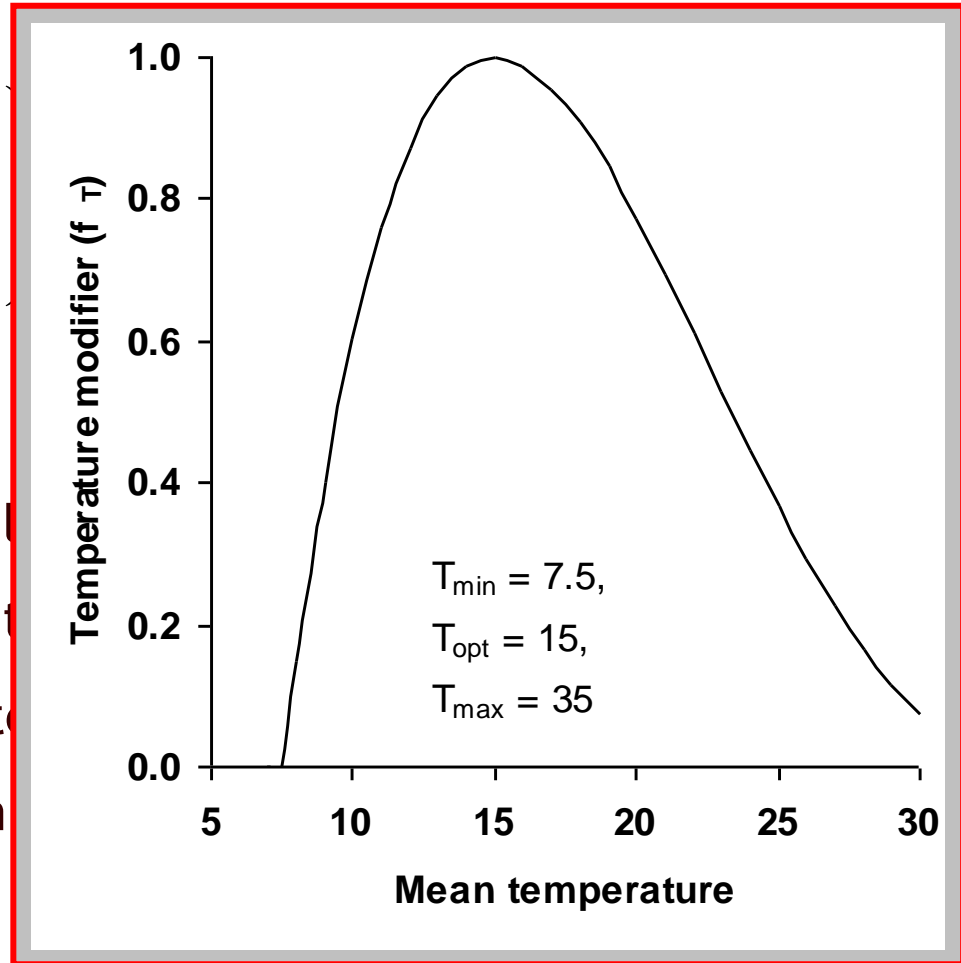
- T_a = mean monthly daily temperature
- T_{min} = minimum temp. for growth
- T_{opt} = optimum temp. for growth
- T_{max} = maximum temp. for growth

Temperature growth modifier

$$f_T(T_a) = \left(\frac{T_a - T_{min}}{T_{opt} - T_{min}} \right)$$

where

- T_a = mean monthly temperature
- T_{min} = minimum temperature
- T_{opt} = optimum temperature
- T_{max} = maximum temperature



Frost growth modifier

$$f_F(d_F) = 1 - k_F(d_F / 30)$$

where

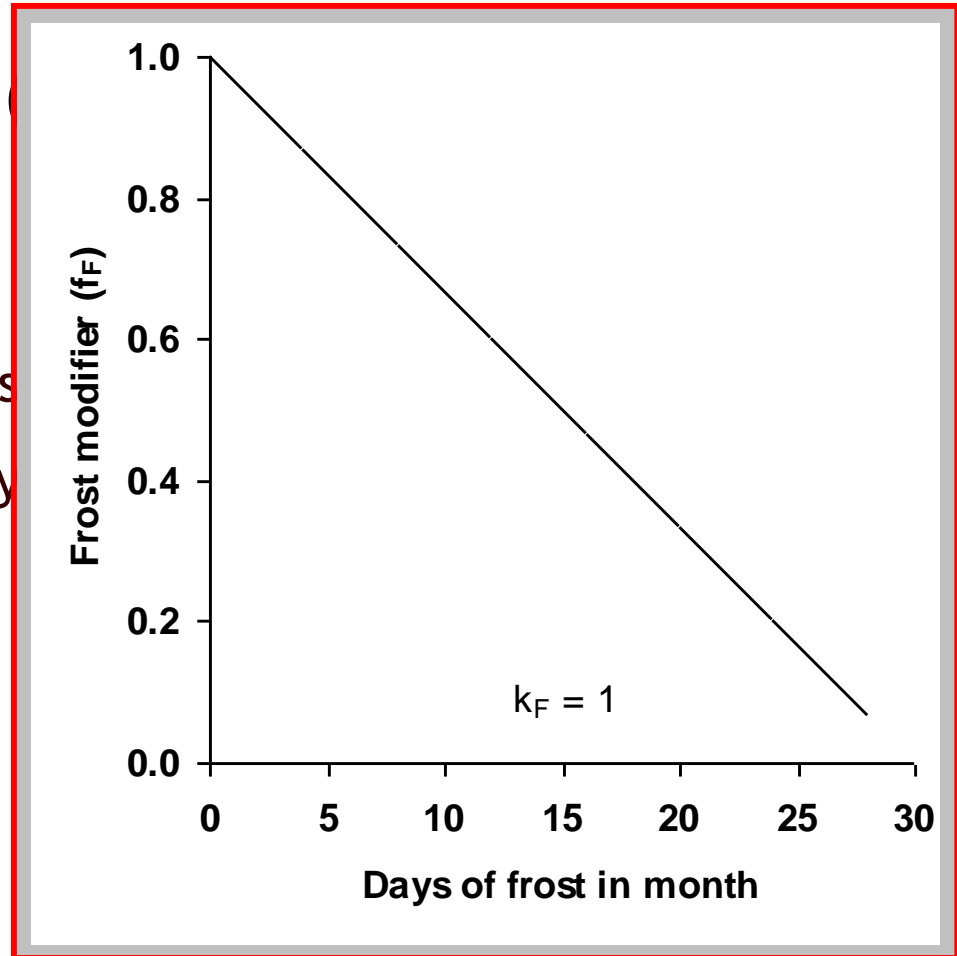
- d_F = number of frosty days in month
- k_F = number of days of production lost for each day of frost

Frost growth modifier

$$f_F(d_F) = 1 - k_F$$

where

- d_F = number of frost days
- k_F = number of days of frost per month



Soil-water growth modifier

$$f_{\theta}(\theta) = \frac{1}{1 + \left[(1 - \theta / \theta_x) / c_{\theta} \right]^{n_{\theta}}}$$

where

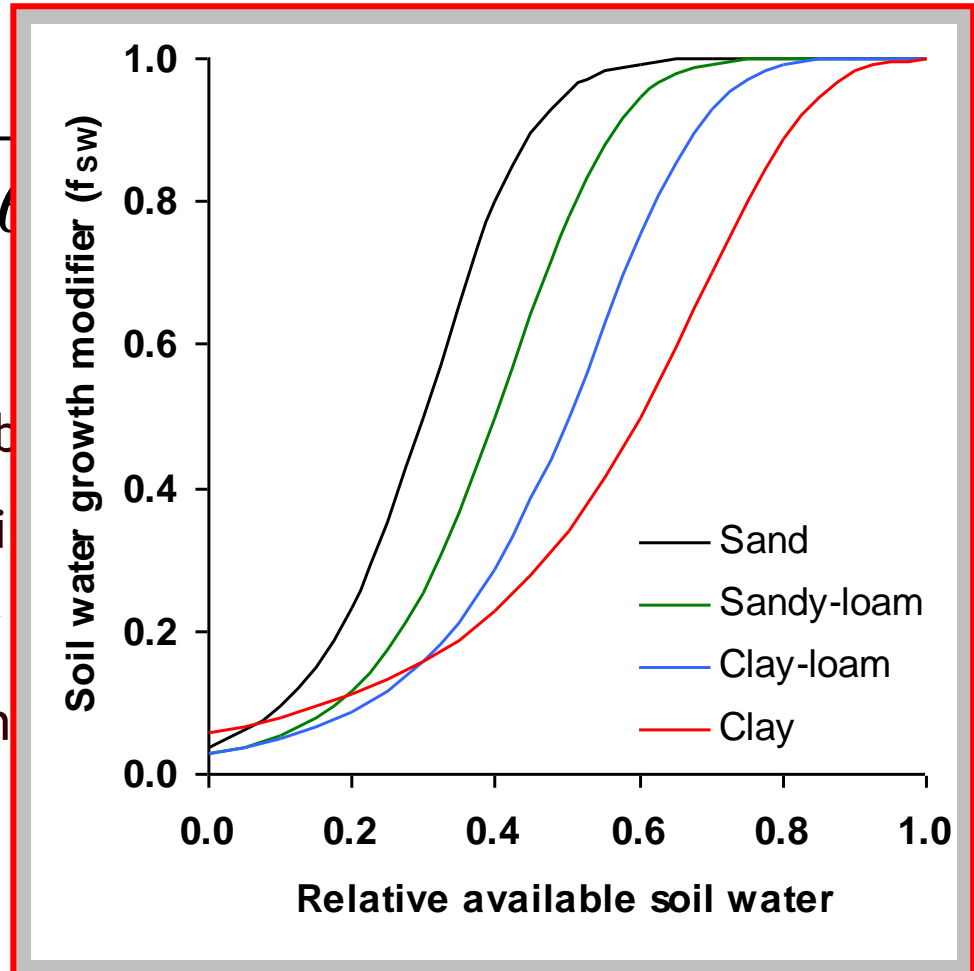
- θ = current available soil water
- θ_x = maximum available soil water
- c_{θ} = relative water deficit for 50% reduction.
- n_{θ} = power determining shape of soil water response

Soil-water growth modifier

$$f_{\theta}(\theta) = \frac{1}{1 + \left[(1 - \theta / \theta_x) / c_{\theta} \right]^{n_{\theta}}}$$

where

- θ = current available soil water
- θ_x = maximum available soil water
- c_{θ} = relative water content
- n_{θ} = power determined by soil texture



VPD growth modifier

$$f_D(D) = e^{-k_D D}$$

where

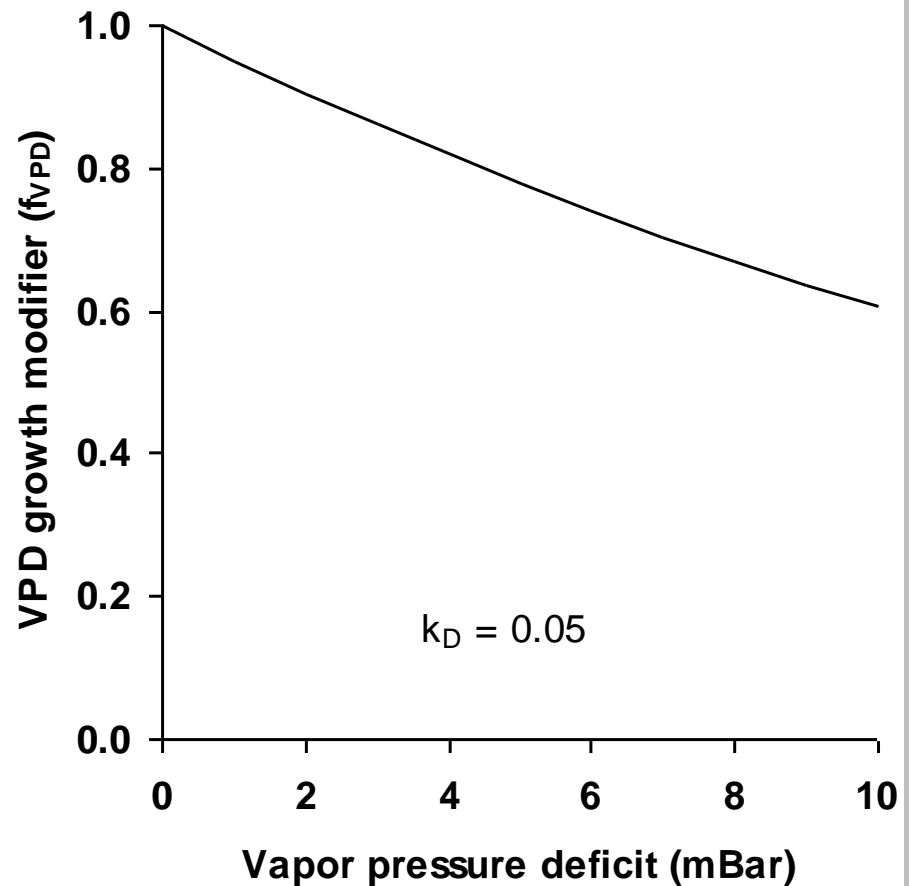
- D = current VPD
- k_D = strength of VPD response

VPD growth modifier

$$f_D(D) = e^{-k_D D}$$

where

- D = current VPD
- k_D = strength of VPD



Age-related growth modifier

$$f_{age}(t) = \frac{1}{1 + (t / r_{age} t_x)^{n_{age}}}$$

where

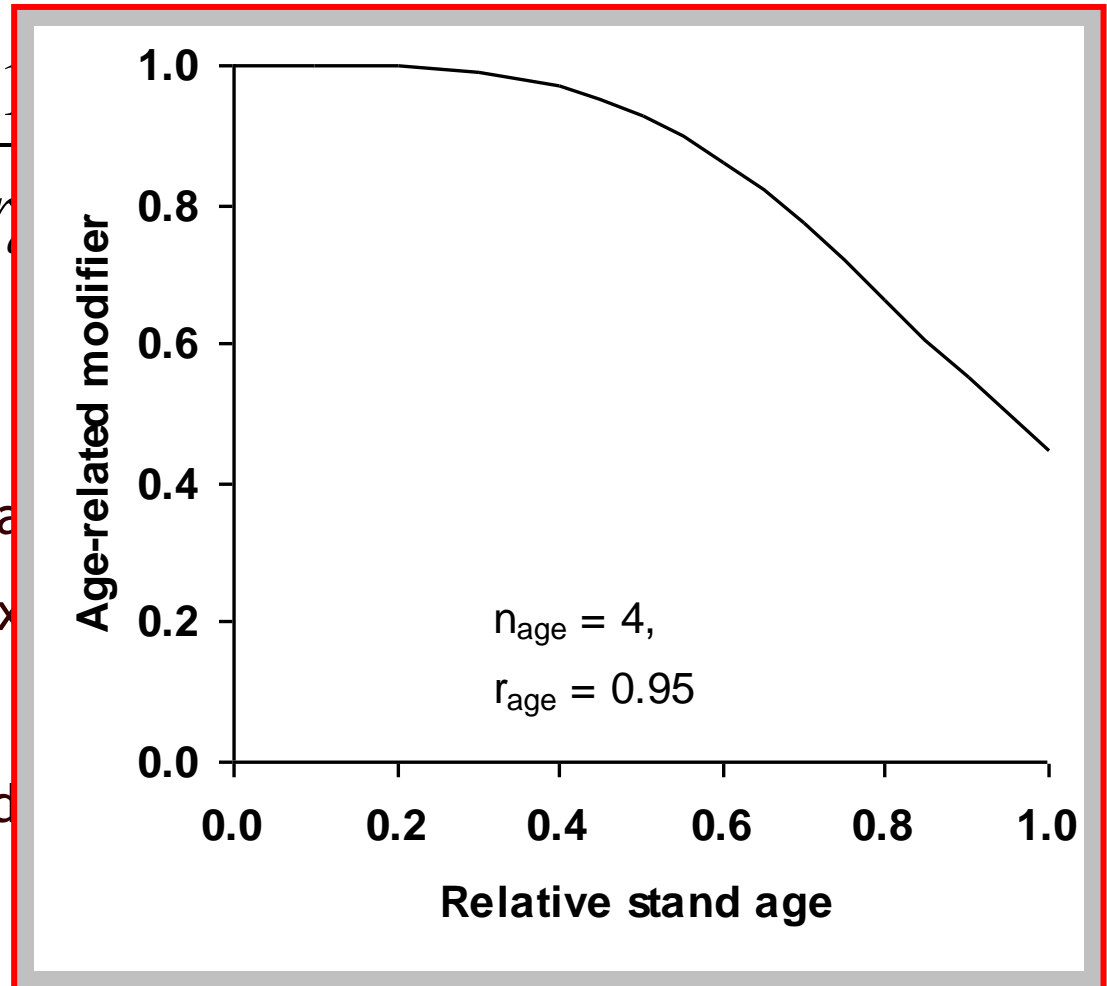
- t = current stand age
- t_x = likely max. stand age
- r_{age} = relative stand age for 50% growth reduction
- n_{age} = power determining strength of growth reduction

Age-related growth modifier

$$f_{age}(t) = \frac{1}{1 + (t/r_{age})^{n_{age}}}$$

where

- t = current stand age
- t_x = likely maximum stand age
- r_{age} = relative stand age at which growth is half of maximum
- n_{age} = power of the age-related modifier



Biomass Partitioning (1/2)

- ✓ NPP is partitioned into biomass pools (tDM ha⁻¹)
 - foliage (W_l)
 - above-ground woody tissue (W_s)
 - roots (W_r)
- ✓ Partitioning rates (η_l , η_r , η_s) depend on site and growth conditions, and stand DBH
- ✓ Litter-fall (γ_l), root-turnover (γ_r) and wood mortality (γ_s) also taken into account

Biomass Partitioning (2/2)

✓ Thus

$$\Delta W_F = \eta_F P_n - \gamma_f W_F$$

$$\Delta W_l = \eta_l P_n - \gamma_l W_l$$

$$\Delta W_s = \eta_s P_n - \gamma_s W_s$$

$$\Delta W_r = \eta_r P_n - \gamma_r W_r$$

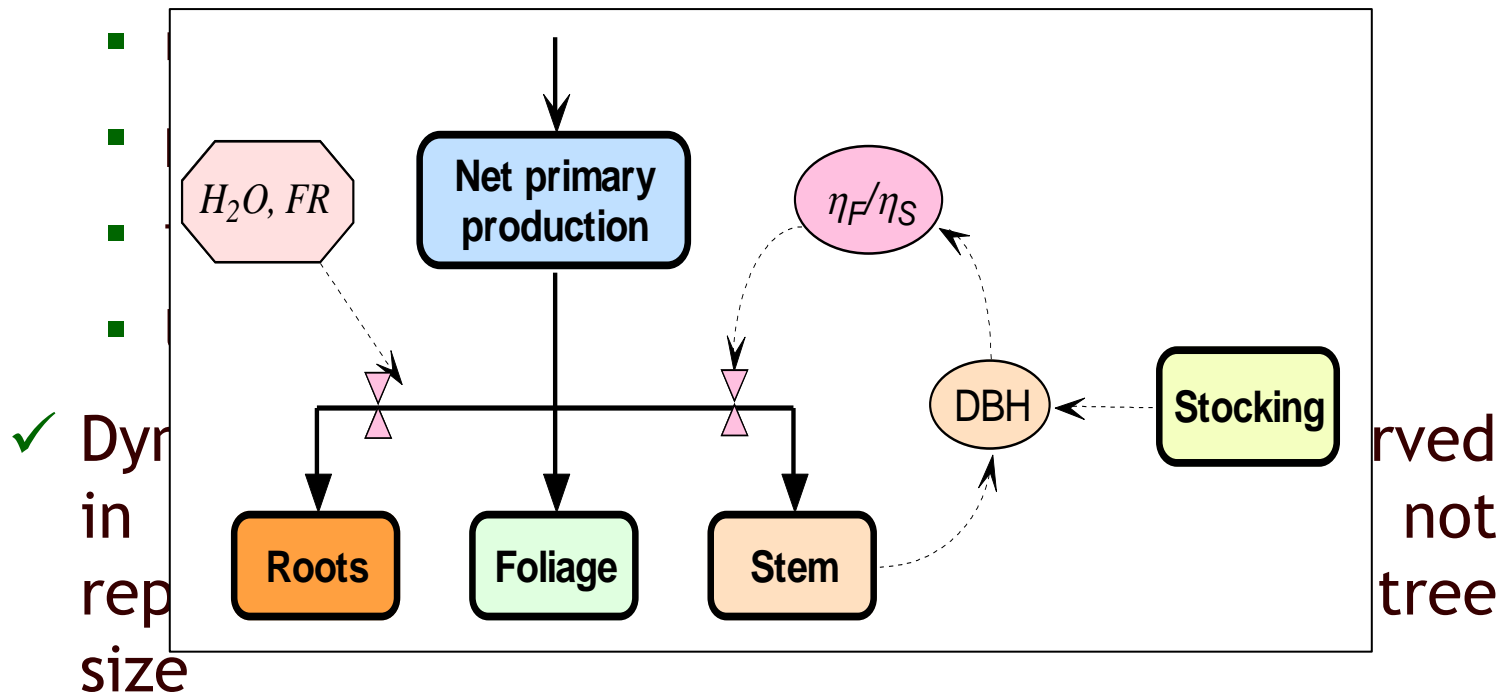
$$\eta_l + \eta_s + \eta_r = 1$$

Allocation in 3-PG

- ✓ A simple-minded approach reproduces well-established responses to site conditions
 - root allocation determined by fertility and ASW
 - poor conditions favour below-ground growth
 - foliage:stem allocation determined by tree size
 - large trees have more allocation to stem wood
- ✓ Dynamic changes in allocation typically observed in thinning or pruning responses are not reproduced because allocation depends on tree size

Allocation in 3-PG

- ✓ A simple-minded approach reproduces well-established responses to site conditions



Root allocation

- ✓ Root allocation is affected by growth conditions through φ and by soil fertility through m

$$\eta_R = \frac{\eta_{Rx}\eta_{Rn}}{\eta_{Rn} + (\eta_{Rx} - \eta_{Rn})m\varphi}$$

where

- $m = m_0 + (1-m_0)FR$
- η_{Rx} = root allocation under poor conditions
- η_{Rn} = root allocation under optimal conditions

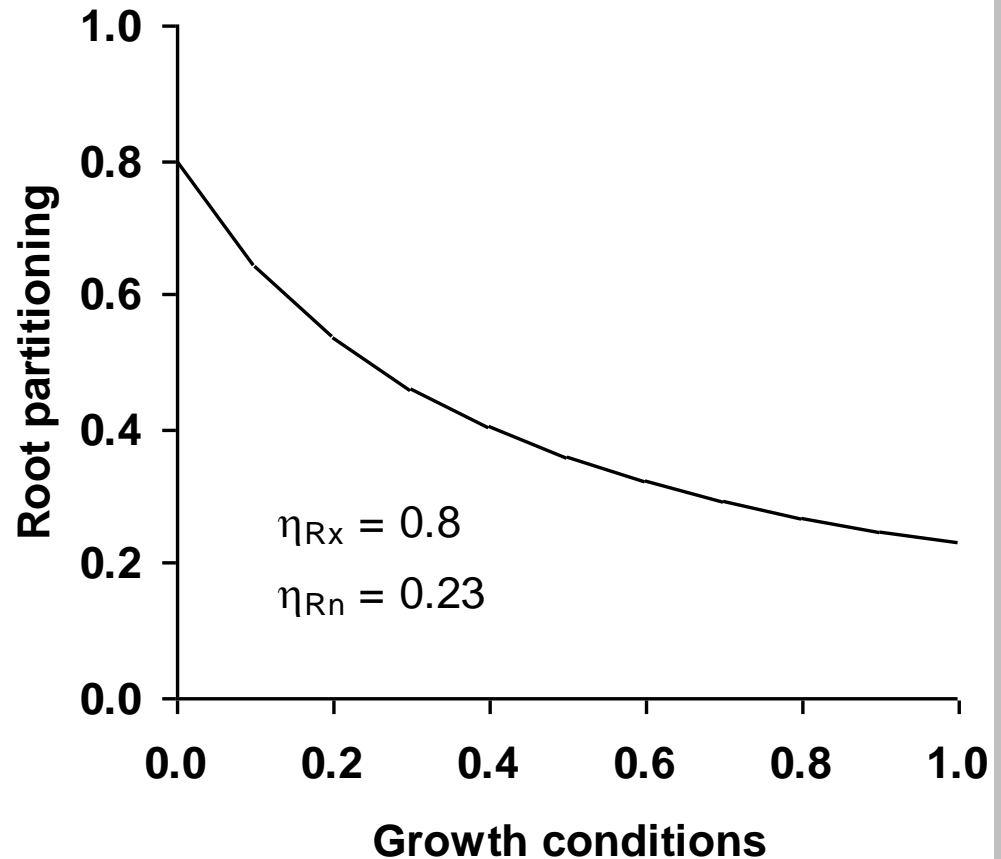
Root allocation

- ✓ Root allocation through ϕ and

$$\eta_R = \frac{\eta_{Rn}}{\eta_{Rn} + (1 - \eta_{Rn}) \phi}$$

where

- $m = m_0 + (1 - m_0) \phi$
- $\eta_{Rx} =$ root allo
- $\eta_{Rn} =$ root allo



Foliage and stem allocation

- ✓ Above-ground allocation is based on foliage:stem partitioning ratio

$$p_{FS} = \eta_F / \eta_S = a_p B^{n_p}$$

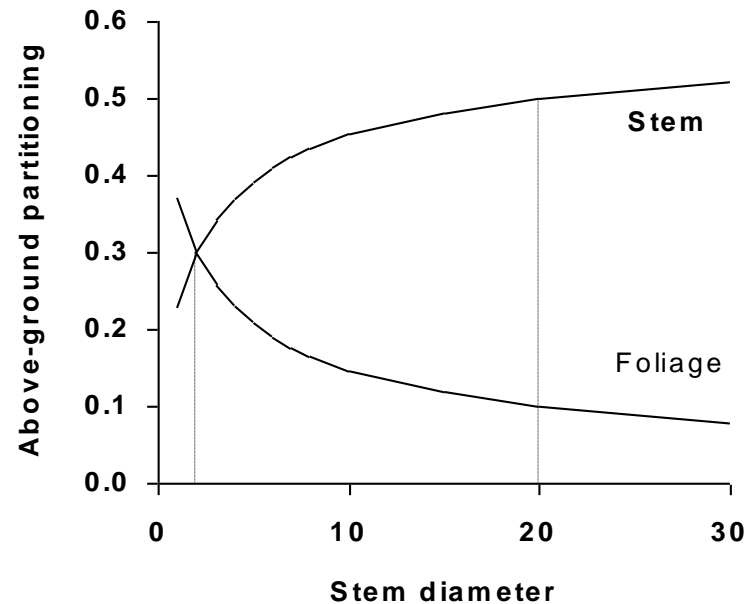
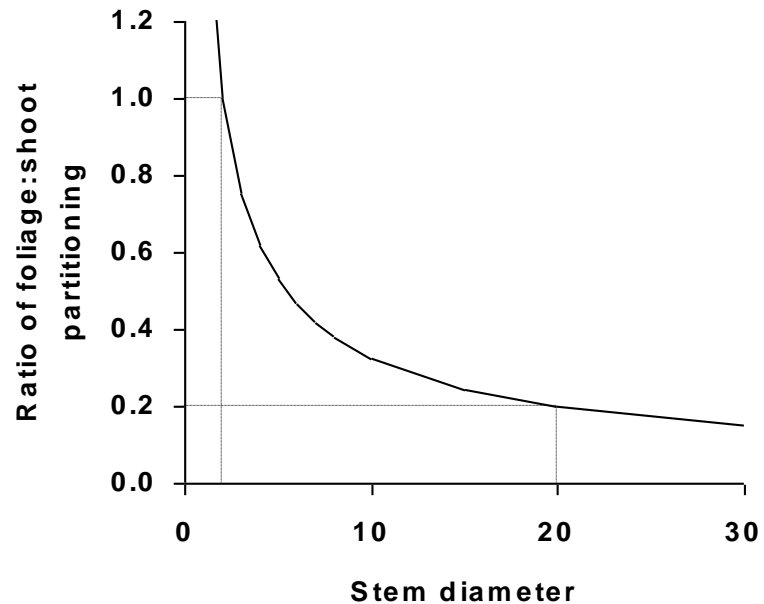
- B is diameter at breast height determined from an allometric relationship between stem mass and B
- a_p , b_p are coefficients determined from pFS at B = 2 and 20 cm

- ✓ Then

$$\eta_S = \frac{1 - \eta_R}{1 + p_{FS}}, \quad \eta_F = p_{FS} \eta_S$$

Tree-size and allocation

- ✓ Increasing DBH decreases foliage allocation and increases stem allocation. Graphs show response when $p_{FS}(2) = 1$, $p_{FS}(20) = 0.2$, $h_R = 0.4$



Litter-fall and root-turnover

- ✓ Litter-fall is an age-dependent fraction of foliage biomass

$$\gamma_F(t) = \frac{\gamma_{Fx}\gamma_{F0}}{\gamma_{F0} + (\gamma_{Fx} - \gamma_{F0})e^{-kt}} \quad , \quad k = \frac{1}{t_{\gamma F}} \ln \left(1 + \frac{\gamma_{Fx}}{\gamma_{F0}} \right)$$

- γ_{F0} = litter-fall rate at age 0
 - γ_{Fx} = maximum litter-fall rate (may be stress-related)
 - $t_{\gamma F}$ = age when $\gamma_F = \frac{1}{2}(\gamma_{F0} + \gamma_{Fx})$
- ✓ Root-turnover is a constant fraction of root biomass ($\gamma_R = 0.015 \text{ month}^{-1}$)

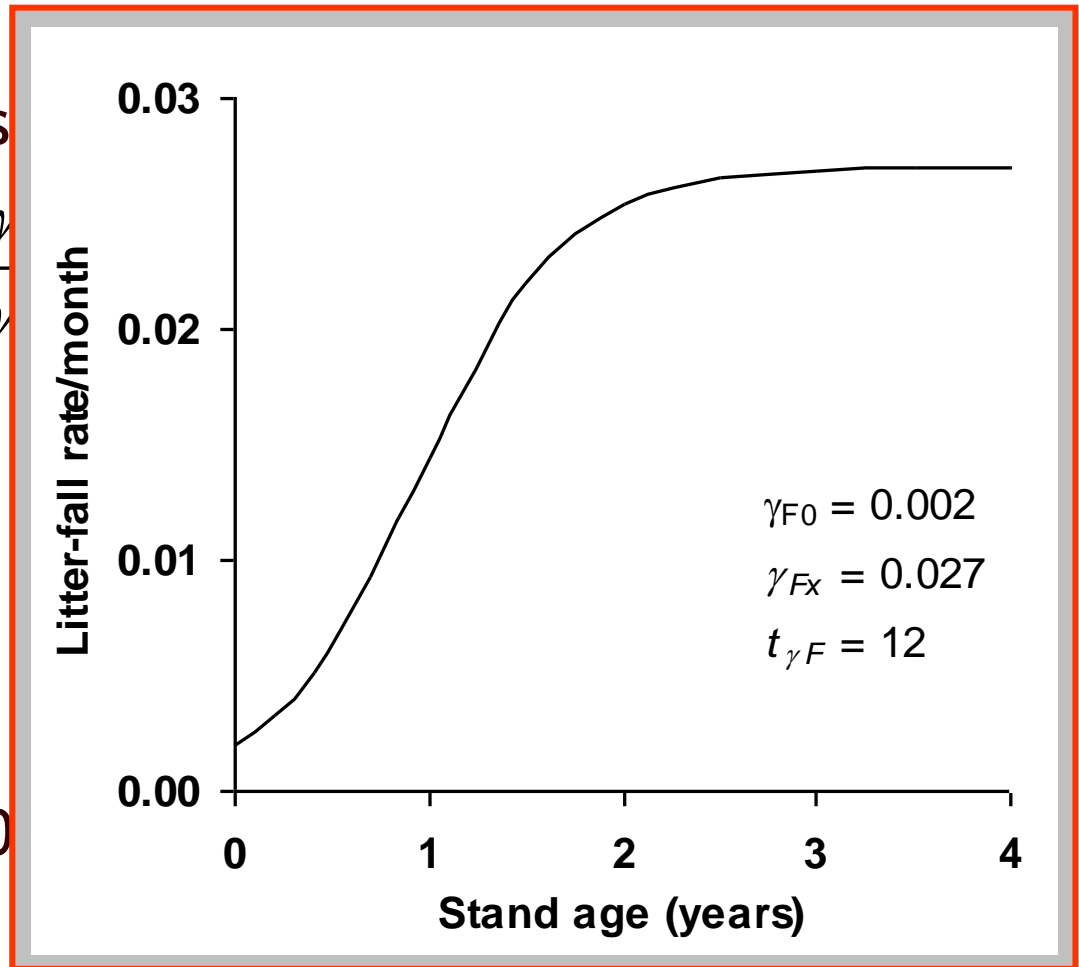
Litter-fall and root-turnover

- ✓ Litter-fall is foliage biomass

$$\gamma_F(t) = \frac{\gamma_{F0} + (\gamma_{Fx} - \gamma_{F0}) \left(1 - e^{-t/t_{\gamma F}}\right)}{1 + e^{-t/t_{\gamma F}}}$$

- γ_{F0} = litter-fall
- γ_{Fx} = maximum
- $t_{\gamma F}$ = age when

- ✓ Root-turnover biomass ($\gamma_R=0.0$)



Summary: the basic C balance equations

- ✓ These are equations for the 3-PG carbon balance submodel
- ✓ includes light interception, assimilation, biomass allocation and mortality
- ✓ α_C and η_R determined from site conditions
- ✓ σ , γ_F , γ_S , γ_R and γ_N possibly age-dependent and/or stress-related

$$P_N = \alpha_C (1 - e^{-k\sigma W_F}) YQ$$

$$\Delta W_F = \eta_F P_N - \gamma_F W_F$$

$$\Delta W_R = \eta_R P_N - \gamma_R W_R$$

$$\Delta W_S = \eta_S P_N$$

$$\Delta N = -\gamma_N N$$

$$\eta_S = (1 - \eta_R) / (1 + p_{FS})$$

$$\eta_F = p_{FS} (1 - \eta_R) / (1 + p_{FS})$$

$$w_s = W_S / N = a_s B^{n_s}$$

$$p_{FS} = a_{FS} B^{n_{FS}}$$

Water balance (1/3)

- ✓ Soil water balance model has a single soil-layer
- ✓ Simple balance between rainfall (R), irrigation (I) and evapotranspiration (E_T)

$$W_+ = \max \{ W_x, W + R + I - E_T \}$$

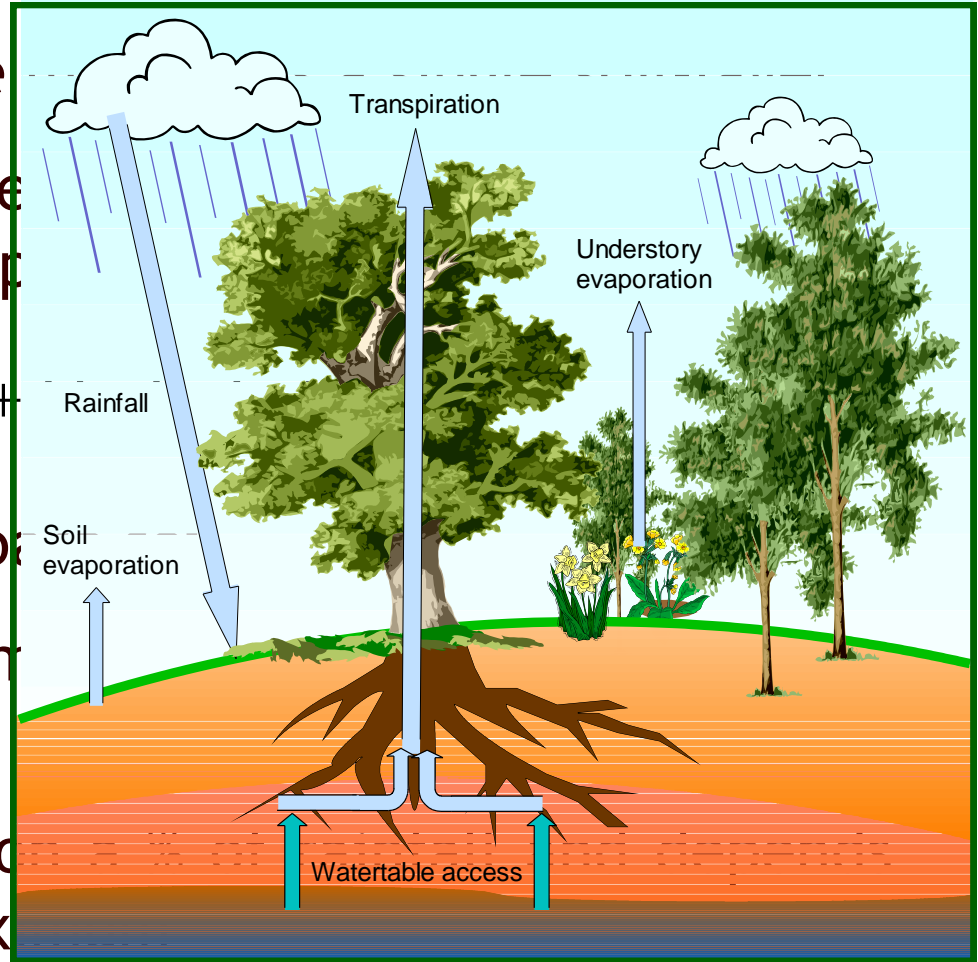
- ✓ No understory or bare soil
- ✓ Excess over maximum storage lost as runoff or drainage
- ✓ Canopy interception a % of rainfall and depends on LAI up to a maximum

Water balance (1/3)

- ✓ Soil water balance
- ✓ Simple balance between precipitation (P) and evapotranspiration (E)

$$W_+ = \max \{ W_x, W + \dots \}$$

- ✓ No understory or bare soil
- ✓ Excess over maximum drainage
- ✓ Canopy interception on LAI up to a maximum



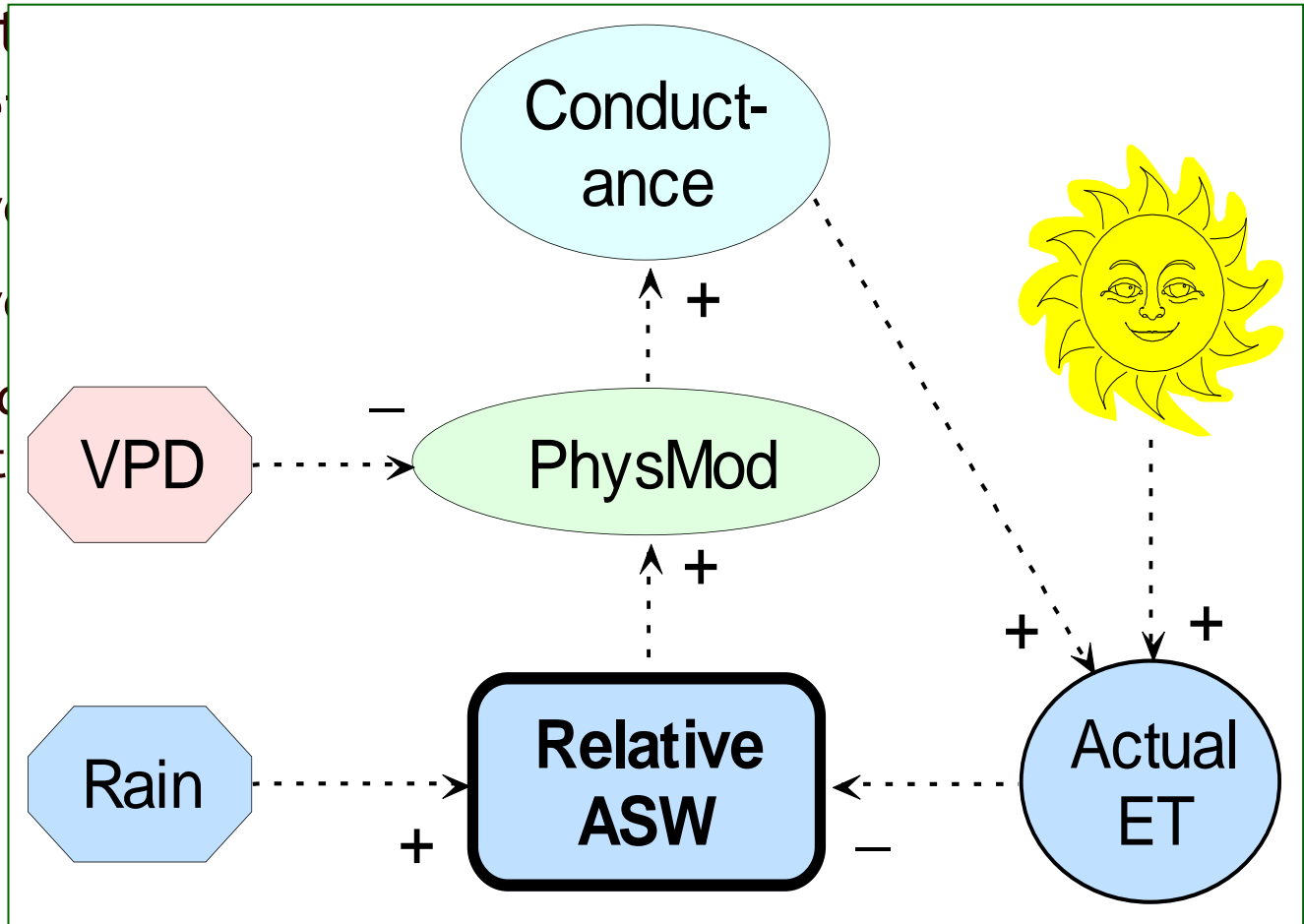
Water balance (2/3)

- ✓ Evapotranspiration determined from a Penman-Monteith equation and canopy conductance
 - driven by incident solar radiation
 - driven by LAI through canopy conductance
 - conductance affected by site and environmental factors through growth modifiers

Water balance (2/3)

✓ Evapotranspiration
Monte Carlo

- driven by VPD
- driven by Relative ASW
- constrained by conductance



Water balance (3/3)

- ✓ Boundary layer conductance is constant (0.2 m s^{-1})
- ✓ Canopy conductance affected by VPD, soil water and stand age through φ , and increases with canopy LAI

$$g_C = g_{C_x} \varphi \min\{L/L_{g_C}, 1\}$$

Where

- $\varphi = \min\{f_{VPD}, f_{SW}\} f_{age}$
- g_{C_x} = maximum canopy conductance
- L_{g_C} = LAI at maximum conductance

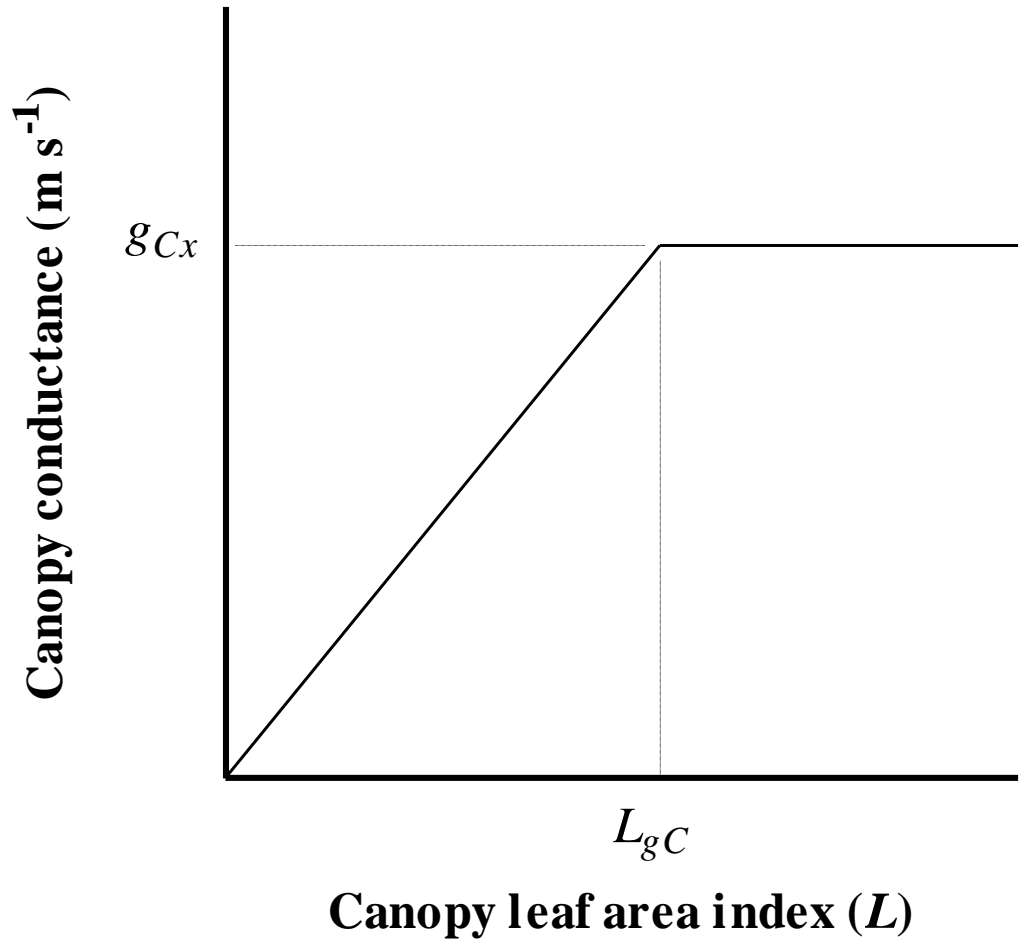
Water balance (3/3)

- ✓ Boundary layer
- ✓ Canopy conductance and stand age canopy LAI

$$g_C = g_{Cx} \varphi \quad \text{m}^2 \text{m}^{-2} \text{s}^{-1}$$

Where

- $\varphi = \min\{fVPD,$
- $g_{Cx} = \text{maximum}$
- $L_{gC} = \text{LAI at max}$

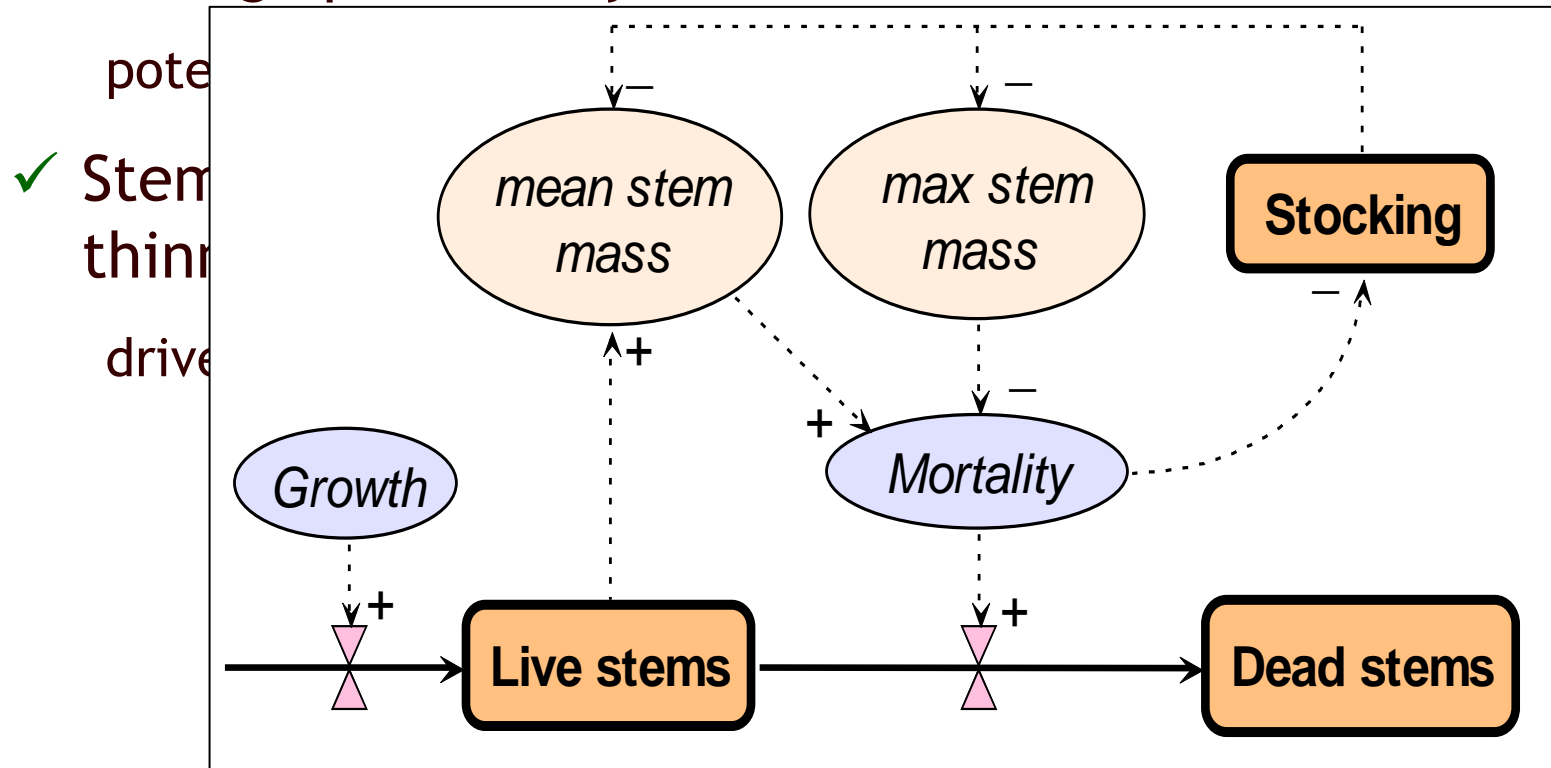


Stem mortality in 3-PG

- ✓ 3-PG includes density independent mortality through probability of death
 - potentially age and stress related
- ✓ Stem mortality in 3-PG is based on the self-thinning law
 - driven by stocking via single-tree stem mass

Stem mortality in 3-PG

- ✓ 3-PG includes density independent mortality through probability of death



Modelling mortality

✓ Two types of mortality:

- density independent
- density dependent or self thinning

✓ Density-independent mortality

- due to random or stress-related effects
- modelled by probability of death γ_N so that

$$\Delta N = - \gamma_N N$$

- γ_N increased in times of stress, e.g. in response to low long-term average f_θ

Self thinning mortality

- ✓ Self thinning-line gives maximum single-tree stem mass (kg/tree) at current stocking

$$w_{Sx}(N) = w_{Sx0}(1000/N)^{3/2}$$

where w_{Sx0} is max. stem mass at 1000 trees ha⁻¹

- ✓ When $w_S > w_{Sx}(N)$, stocking is reduced. Thus
 - at 1: no mortality

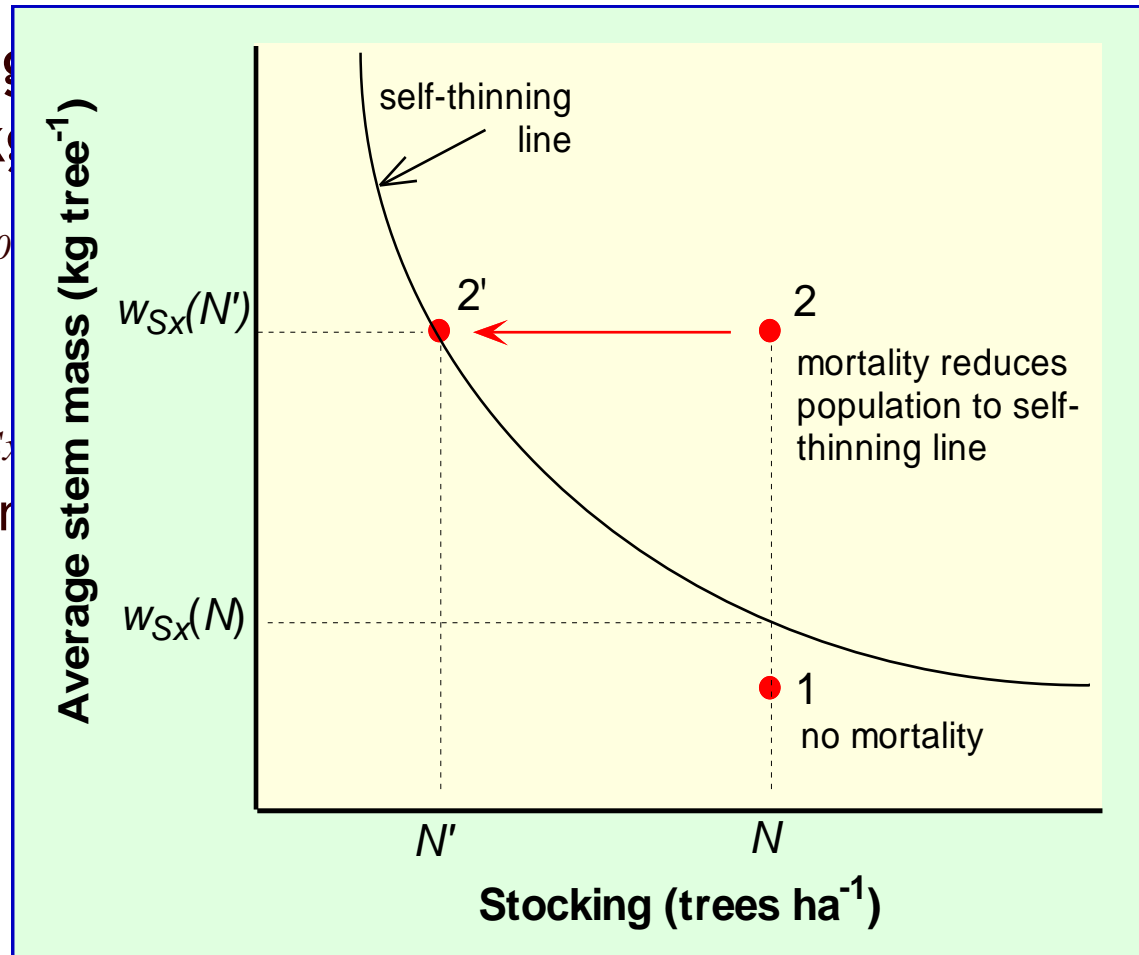
Self thinning mortality

- ✓ Self thinning
- stem mass (kg tree⁻¹)

$$w_{Sx}(N) = w_{Sx0}$$

where w_{Sx0} is

- ✓ When $w_S > w_{Sx}$
 - at 1: no mortality



Calculation of stem volume

- ✓ Stem volume calculated either from
 - allometric relationship with stand DBH,
 - or from wood density using

$$V = (1 - p_{BB})W_S/\rho$$

where p_{BB} is fraction of stem biomass in branch and bark and ρ is stem density

- ✓ Note that p_{BB} and ρ can vary widely across a site and calculation of stem volume from W_S can be error prone

Final comments

- ✓ Some areas are not covered in detail, e.g.
 - details of Penman-Monteith equation
 - rainfall interception by canopy
 - an attempt to account for partial canopies (which does not work well)
- ✓ However, the above gives a good coverage of the basics of 3-PG



3PG

An aerial photograph of a large-scale plantation, likely a rubber or coffee plantation, showing neat rows of trees and winding dirt roads. A prominent white starburst graphic is centered in the image, containing the text "The end!".

**The
end!**